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A Tachyonic Gluon Mass: between Infrared and Ultraviolet

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Abstract

The gluon spin coupling to a Gaussian correlated background gauge field induces an effective tachyonic gluon mass. It is momentum dependent and vanishes in the UV only like $1/p^2$. In the IR, we obtain stabilization through a positive $m^2_{\rm conf}(p^2)$ related to confinement. Recently a purely phenomenological tachyonic gluon mass was used to explain the linear rise in the $q\bar{q}$ static potential at small distances and also some long standing discrepancies found in QCD sum rules. We show that the stochastic vacuum model of QCD predicts a gluon mass with the desired properties.

Recently it was argued [1, 2] that an effective negative gluon (mass)^2 produces the linear rise of the static quark-antiquark potential observed in lattice calculations [3] at rather short distances below the nonperturbative IR scale of QCD. This can also be related to an unconventional Q^{-2} piece in the running QCD coupling $\alpha_s(Q^2)$ [4, 5] indicated again by lattice data on the gluon condensate and the 3-gluon vertex [6]. Later in ref. [7], it was shown that a linear term in the potential for small distances appears in the evaluation of the Wilson loop integral in a Gaussian correlated background gauge field if the paramagnetic (spin related) gluon coupling is taken into account. A gluon propagator with a nonperturbative tachyonic mass in the evaluation of the operator product expansion for QCD sum rules also solves old discrepancies in the pion and scalar gluonium channels [2].

In ref. [2], the view was taken that a gluon mass is not just a convenient way to parametrize nonperturbative infrared (IR) effects, but that a tachyonic gluon mass also enters into the basic short distance operator product expansions. In this letter, we propose less radically that a tachyonic gluon mass does appear at momenta above the QCD scale, although at very high momenta/small distances it goes to zero. Thus, in the real ultraviolet regime (UV), the basic perturbative operator product expansion should be valid.* Of course, if one discusses an operator product expansion in an effective theory including a tachyonic gluon mass in which very high momenta are already integrated out, this mass shows up in the expansion. However, at very small distances, the expansion loses its basis.

In the IR, a tachyonic gluon mass would cause divergences. Thus there should be an overcompensating positive effective gluon (mass)^2. In the case of the hot electroweak theory close to the symmetric phase, we have argued earlier [9] for the following picture: a tachyonic gauge boson mass related to spin-spin forces is effective at intermediate momenta and a positive (mass)^2 related to the area law of confinement is dominant in the gauge boson propagator in the IR. Our equations in ref. [9] were written for a general dimension d and gauge group $SU(N)$, but we had restricted our discussion to the case $d = 3$ and $N = 2$, which is the relevant one for the discussion of the electroweak phase transition. In this letter, we analyze the QCD case $d = 4$ and $N = 3$.

*Concerning a $1/Q^2$ behavior consider the following simple function: $(\alpha Q^4 + \beta Q^2)^{-1}$. If $\beta \gg \alpha$ is large, it behaves like $\frac{1}{\beta Q^2}$ for $Q^2 < \beta/\alpha$, i.e. for rather large Q^2!

Our actual calculations use the "stochastic vacuum model" [10] for the QCD vacuum. In brief, this model provides the area law of confinement in a very simple and intuitive way. It describes the QCD vacuum by a stochastic process and assumes that only Gaussian correlations are present in the cumulant expansion. The nonperturbative part of the gauge invariant field strength correlator is given by (after contracting the Lorentz indices):

$$\langle\langle g^2 F^a_{\mu\nu}(x', x_0) F^b_{\mu\nu}(x, x_0)\rangle\rangle = \delta^{ab}\langle g^2F^2\rangle D\left(\frac{(x'-x)^2}{a^2}\right). \tag{1}$$

Here $\langle g^2F^2\rangle \equiv \langle g^2 F^a_{\mu\nu}F^a_{\mu\nu}\rangle$ is the usual "local" gluon condensate, and a denotes the correlation length. The tensor $F_{\mu\nu}(x, x_0)$ is the ordinary field strength tensor $F_{\mu\nu}(x)$ parallel-transported to a fixed common reference point x_0 [10] and a Euclidean metric is used. A typical form of D used in lattice evaluations [11] is $\exp(-|x'-x|/a)$.

The inverse gluon propagator (in the Feynman background gauge) in a background gauge field reads

$$K^{ab}_{\mu\nu} = \left[-\mathbb{D}^2\delta_{\mu\nu} + m^2\mathbf{1}_c\delta_{\mu\nu} + 2ig\mathbb{F}_{\mu\nu}\right]^{ab}. \tag{2}$$

Here we have allowed for the presence of a gluon mass for future convenience. Linearized gauge field excitations $a^b_\mu(x)$ interact with the background field via the Lagrangian $\frac{1}{2}a^b_\mu K^{bc}_{\mu\nu}a^c_\nu$. We have to distinguish the "diamagnetic" coupling via the ordinary minimal-substitution term $\mathbb{D}^2\delta_{\mu\nu}$ and a non-minimal "paramagnetic" coupling via the $2igF_{\mu\nu}$ term. In [9] we used this interaction Lagrangian in order to compute the "mass operator" Σ of the gluon (vacuum polarization) to lowest nontrivial order in $\langle g^2F^2\rangle$. It is defined such that

$$G = [G^{(0)-1} - \Sigma]^{-1} \tag{3}$$

where $G^{(0)}$ and G are the free and the dressed propagator of the gluon, respectively. The latter describes the propagation of a perturbative gluon in the background of a Gaussian-correlated random Yang-Mills field. Taking $x_0 = (x+x')/2$ as the reference point, the result for Σ is diagonal in the Lorentz and the color indices:

$$\Sigma(x,x')^{ab}_{\mu\nu} = \mathcal{S}(z^2 \equiv (x-x')^2)\delta_{\mu\nu}\delta^{ab}. \tag{4}$$

The function $\mathcal{S}(z^2) \equiv \mathcal{S}_F(z^2) + \mathcal{S}_A(z^2)$ receives contributions from two rather different physical mechanisms: $\mathcal{S}_F$ results from the spin-related paramagnetic

interactions via the $F_{\mu\nu}$ term alone, while $\mathcal{S}_A$ stems from diagrams involving the diamagnetic coupling and coincides with the result that we would obtain for spin-0 bosons. Fourier-transforming with respect to z, we obtain $\tilde{\mathcal{S}}(p^2, m^2) \equiv \tilde{\mathcal{S}}_F + \tilde{\mathcal{S}}_A$ with†

$$\begin{aligned} \tilde{\mathcal{S}}_F(p^2, m^2) &= \frac{3}{32\pi^3}\langle g^2F^2\rangle \int_0^\infty dq q^3 \tilde{D}(q^2) \int_{-1}^{+1} \frac{d\cos\vartheta (1-\cos^2\vartheta)^{1/2}}{(p^2+q^2+m^2-2pq\cos\vartheta)} \\ &= \frac{3}{128\pi^2}\langle g^2F^2\rangle \int_0^\infty dq q^3 \tilde{D}(q^2) \times \\ & \quad \frac{1}{(pq)^2}\left[(p^2+q^2+m^2) - \sqrt{(p^2+q^2+m^2)^2-(2pq)^2}\right] \end{aligned} \tag{5}$$

and similarly for $\tilde{\mathcal{S}}_A(p^2, m^2)$.

In view of eq. (3) it is natural to interpret $m^2_{\rm tach}(p^2) \equiv -\tilde{\mathcal{S}}(p^2, m^2 = 0)$ as the effective, momentum-dependent mass of a gluon propagating in a stochastic gauge field background. One finds that $\tilde{\mathcal{S}}(p^2, 0) > 0$ so that $m^2_{\rm tach} < 0$, i. e. the effective mass is *tachyonic*. Its value at $p^2 = 0$, $m^2_{\rm tach} \equiv m^2_{\rm tach}(0)$, can be written down in closed form‡:

$$-m^2_{\rm tach} = \tilde{\mathcal{S}}(p^2 = 0, m^2 = 0) = \frac{3}{32}a^2\langle g^2F^2\rangle\{1+\delta\}\int_0^\infty dw\ D(w) \tag{6}$$

Here the "1" inside the curly brackets on the RHS of eq. (6) comes from the paramagnetic $\mathcal{S}_F$ while

$$\delta = \frac{17}{12} - \frac{4}{3}\ln 2 \approx -0.492$$

is the contribution of the diamagnetic piece $\mathcal{S}_A$. We observe that the tachyonic nature of the induced mass is a consequence of the paramagnetic spin interaction which dominates over the diamagnetic one. The latter generates a (mass)^2 which is numerically smaller and of opposite sign.

It is quite remarkable that, up to a constant factor, the RHS of eq. (6) equals precisely the well-known [10] result of the stochastic vacuum model for the string tension (with the color sources in the fundamental representation of $SU(3)$):

$$\sigma_{\rm fund} = \frac{\pi^2}{144}a^2\langle g^2F^2\rangle \int_0^\infty dw D(w). \tag{7}$$

†See eq. (A.34) of [9].

‡Cf. eq. (A.38) of [9] where $w \equiv z^2/a^2$.

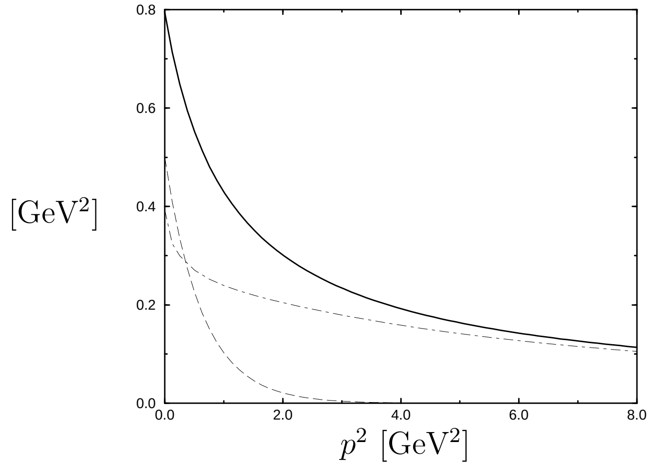

Figure 1: $\mathcal{S}_F(m^2_{\rm conf}=0)$ (solid line), $\mathcal{S}_F(m^2_{\rm conf}\neq 0)$ (dashed-dotted line) and $m^2_{\rm conf}$ (dashed line) as function of p^2.

Thus the gluon mass can be expressed directly in terms of the string tension:

$$-m^2_{\rm tach} = \frac{18}{\pi}(\ln 2 - \frac{5}{16})\sigma_{\rm fund} \approx 2.18\sigma_{\rm fund}. \tag{8}$$

It is a universal prediction of the stochastic vacuum model in the sense that no separate knowledge of the gluon condensate, the correlation length and the shape of the structure function D is needed. In fact, in eq. (8) we can directly use the experimental value for the string tension. From the charmonium spectrum one obtains [13] $\sigma_{\rm fund} \approx (430 MeV)^2$ which leads to $m^2_{\rm tach} \approx -0.40 (GeV)^2$ or $|m_{\rm tach}| \approx 635 MeV$.

This is indeed a tachyonic mass of the order of magnitude postulated in ref. [2] on phenomenological grounds. Since to lowest order

$$G^{ab}_{\mu\nu}(p^2) = \left[\frac{1}{p^2} - \frac{m^2_{\rm tach}}{p^4} + \mathcal{O}\left(\frac{1}{p^6}\right)\right]\delta_{\mu\nu}\delta^{ab}, \tag{9}$$

with an $1/p^4$ correction to the free propagator, we see that in the static $q\bar{q}$ potential $V(r)$ the tachyonic gluon mass gives rise to a term that grows linearly with r. However, in contrast to the asymptotic ($r \to \infty$) regime, in which $V(r) \approx \sigma_{\rm fund} r$ dominates the Coulomb term, we are at much shorter distances here where the linear term is only a small correction to the $1/r$ law.

In fig. 1, we present $\tilde{\mathcal{S}}_F(p^2, m^2=0)$ for $D(w) = \exp(-|z|/a) = \exp(-|w|^{\frac{1}{2}})$ and $a = 0.22$fm [11] as a function of p^2. It vanishes for large p^2 rather slowly

as

$$\tilde{\mathcal{S}}_F(p^2) \underset{p^2 \gg \frac{1}{a^2}}{\longrightarrow} \frac{3}{8}\frac{\langle g^2F^2\rangle}{p^2}. \tag{10}$$

Eqs. (5), (8) and (10) are our main results. The "paramagnetic dominance" and the resulting tachyonic sign of the gluon $(\text{mass})^2$ are related to the well known instability of the Savvidy vacuum [14]. The latter models the true QCD vacuum in terms of an external gauge field with a constant field strength. While already lower in energy than the perturbative vacuum, it is unstable and decays towards the true ground state precisely because the destabilizing paramagnetic interaction of the vacuum fluctuation overrides the diamagnetic one. The paramagnetic dominance encountered above (and discussed in detail in ref. [9]) is a remnant of this phenomenon for non-constant backgrounds.

The self-energy Σ can also be used in order to understand why, first of all, the perturbative vacuum is unstable. Inserting Σ, which is proportional to $\langle g^2F^2\rangle$, into a perturbative gluon loop, we obtain the $\langle F^2\rangle$ term of the 1-loop effective potential for the condensate, $V_{\text{eff}}(\langle F^2\rangle)$. This term turns out to be negative, i. e. $\langle g^2F^2\rangle = 0$ is not the minimum of V_{eff}, and a non-zero condensate tends to form. But, in contrast to the $d = 3$ case, for $d = 4$, the term that is linear in $\langle F^2\rangle$ is logarithmically divergent in the UV.

In ref. [9], we also calculated the standard perturbative 1-loop gluon self-energy Π and we introduced the $F_{\mu\nu}$ correlator (1) only in a second step (fig. 2). This should be an equivalent procedure and indeed, in three dimensions, this can be easily checked. For $d = 4$, the perturbative vacuum polarization $\Pi(q^2)$ (fig. 2) is divergent. It has to be renormalized at some Euclidean $q^2 = \mu^2$ in the perturbative range $\mu^2 > 1/a^2$.[§] Taking only the contribution from the non-minimal gauge boson coupling, one obtains (see eq. (2.20) of ref. [9])

$$\Pi_F^{\text{ren}}(p^2,\mu^2) = \frac{4}{(4\pi)^2}\log\frac{p^2}{\mu^2} \tag{11}$$

which is to be inserted into

$$V_{\text{eff}} = \frac{3}{4}\langle g^2F^2\rangle \int \frac{d^4p}{(2\pi)^4}\Pi_F^{\text{ren}}(p^2,\mu^2)\tilde{D}_{\text{eff}}(p^2). \tag{12}$$

[§]According to ref. [12] the stochastic vacuum model describes the QCD vacuum at a renormalization scale μ not much above $1/a$.

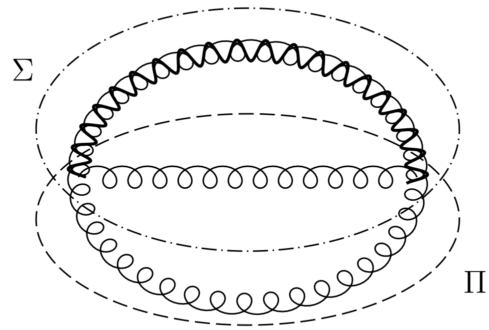

Figure 2: First order contribution of the correlator (double wiggled) to the gluon loop

This is indeed negative if $\mu^2 > 1/a^2$, the typical scale set by $\tilde{D}$.

The 1-loop expression resulting from the Σ approach,

$$V_{\rm eff} \propto \int \frac{d^4p}{(2\pi)^4} \{\log(p^2 - \tilde{\mathcal{S}}_F(p^2)) - \log p^2\}, \qquad (13)$$

with an appropriate constant already subtracted, is still logarithmically divergent. Obviously the $1/p^2$ decay of $\tilde{\mathcal{S}}(p^2)$ is too slow to make Σ completely irrelevant in the UV. At first sight, this seems to be an UV-effect related to the gluon mass similar to the one discussed in ref. [2]. However, our divergent term is proportional to the stochastic average of the classical action $\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$ and can be removed by the same renormalization as in perturbation theory. This is an important difference as compared to ref. [2]. Note also that in contrast to the case with a constant background field, we are not plagued by IR singularities any longer.

To really obtain an instability at $\langle F^2\rangle = 0$, the positive tree term has to be dominated by the quantum corrections. The 1-loop potential could be further improved by including Wilson type renormalization effects [15]: the effective $g^2(k^2)$ is increasing in the infrared direction $k^2 \to 0$.

Finally, let us discuss the gluon propagator and the stochastically averaged 1-loop effective potential

$$V_{\rm eff}(\langle F^2\rangle) = \frac{1}{2}\langle\langle {\rm Tr}\ \log(K)\rangle\rangle \qquad (14)$$

beyond the linear approximation in $\langle F^2 \rangle$. The effective potential can be represented in terms of a world-line path integral [9] over closed paths $y(\tau)$. Its integrand contains the pivotal factor

$$\langle\langle \mathrm{tr}_{\mathrm{cL}} P \exp ig \int d\tau (\dot{y}_\mu A_\mu(y) + 2F(y))\rangle\rangle .$$

Apart from the paramagnetic F-term discussed above, this is precisely the stochastic average of the Wilson loop operator which signals confinement. For large loops, it behaves as $\exp(-\sigma \mathcal{A}[y]T)$ where $\sigma \equiv \sigma_{\mathrm{fund}} \propto \langle g^2 F^2 \rangle$ is the string tension (7) and $\mathcal{A}[y]$ denotes the area of the minimal surface bordered by $y(\tau)$. This is one of the most important results of the stochastic vacuum model. Thus the contribution to V_{eff} which originates from the "diamagnetic" interaction of very large loops is proportional to

$$V_{\mathrm{eff}}^{\mathrm{conf}} = -\frac{1}{2}\int_0^\infty \frac{dT}{T} T^{-2} \int Dy \exp(-\int_0^1 d\tau \dot{y}^2/4)\{\exp\left(-\sigma \mathcal{A}[y] T f(\mathcal{A}[y]T)\right) - 1\}. \tag{15}$$

Here we use appropriately rescaled dimensionless variables. The path integration is subject to the conditions $y(0) = y(1)$ and $\int_0^1 y(\tau) d\tau = 0$. In principle, the cut-off function f with $f(\infty) = 1$ and $f(0) = 0$ could be calculated from the model; it serves the purpose of cutting out the contribution of small loops so that the perturbative expression at small distances is not changed. Using the identity $(4\pi T)^{-2} = \int \frac{d^4p}{(2\pi)^4} \exp(-Tp^2)$, the confinement-related part of V_{eff} assumes the form¶

$$V_{\mathrm{eff}}^{\mathrm{conf}}(\langle F^2 \rangle) = \int \frac{d^4p}{(2\pi)^4} I(p^2) \tag{16}$$

where

$$I(p^2) = -4\pi \int_{-\infty}^{+\infty} d\omega F(\omega) \int_0^\infty \frac{dT}{T^2} \exp(-p^2 T) G\left(\frac{\omega}{T}\right) \tag{17}$$

with

$$F(\omega) = \int Dy \exp\left\{ i\mathcal{A}[y]\omega - \int_0^1 d\tau \dot{y}^2/4 \right\} \tag{18}$$

and

$$G(x) = 2\int_0^\infty d\bar{A} \cos(\bar{A}x) \left[\exp\{-\sigma \bar{A} f(\bar{A})\} - 1\right]. \tag{19}$$

¶One of the authors (M. G. Schmidt) thanks M. Laine for a helpful discussion on that point.

Expression (17) can be written in the form $\{\ln(p^2 + m^2_{\text{conf}}(p^2)) - \log p^2\}$. Eq. (16) suggests that this defines an effective IR mass $m^2_{\text{conf}}(p^2)$ due to confinement [9]. Note that the counterterm cancels in this relation.

The evaluation of (17), (18) and (19) requires numerical studies, in particular of the most interesting function $F(\omega)$ [16]. The main scale in this problem is the string tension σ; thus we expect $m^2_{\text{conf}}(p^2 = 0) \sim \sigma$. The shape of the cut-off function $f(\bar{A})$ determines the profile in p^2. At $p^2 = 0$, m^2_{conf} should dominate the negative $-\tilde{\mathcal{S}}_F(p^2 = 0)$ in order to obtain a reasonable IR behavior (though interferences between confinement and spin effects are not treated carefully in the approach presented above).

The confinement mass $m^2_{\text{conf}}(p^2)$ should be also included in our calculation of $\tilde{\mathcal{S}}_F$. In a first approximation, one could substitute it as an m^2 in formula (5) (of course, it is again not an ultraviolet mass requiring regularization). This lowers $\tilde{\mathcal{S}}_F$ and the result depends a lot on the balance between m^2_{conf} and $-\tilde{\mathcal{S}}_F$ at small p^2. In fig. 1, we have included a plausible function $m^2_{\text{conf}}(p^2)$ to demonstrate the effect. We expect $m^2_{\text{conf}}(p^2)$ to dominant in the IR, i. e. for $p^2 \lesssim 1/a^2$. The range of $\tilde{\mathcal{S}}_F(p^2)$ extends to higher p^2. It vanishes only like $1/p^2$. Thus $-\tilde{\mathcal{S}}_F(p^2)$ mimics a tachyonic UV mass.

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